

Power System Analysis-2

BEE703(IPCC)

Prepared by

Dr. Raghavendra L

Associate Professor

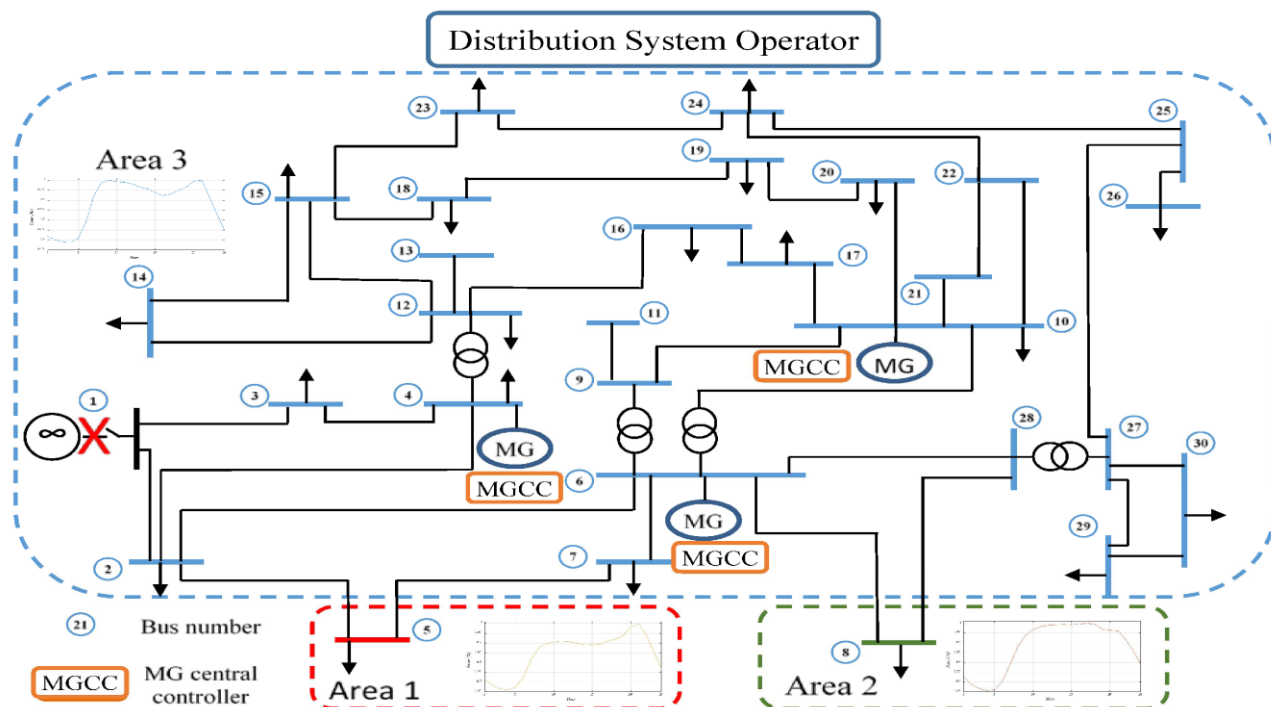
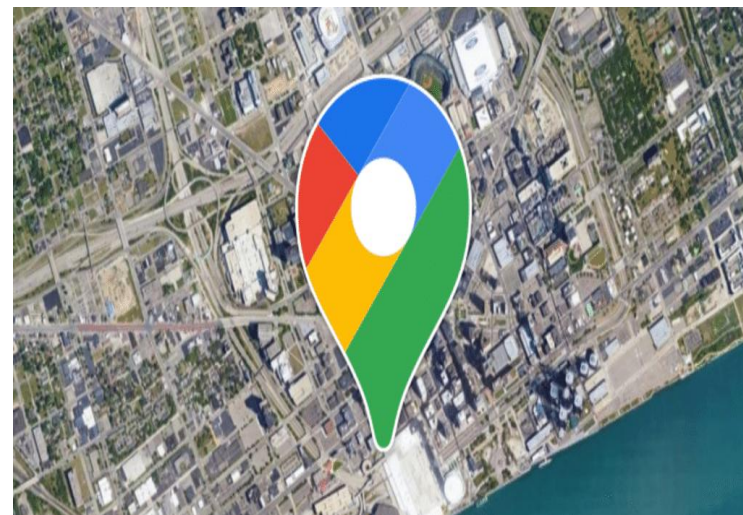
Dept of EEE, ATMECE

1. Introduction
2. Basic definitions of Elementary graph theory
3. Tree, cut-set, loop.
4. Formation of Incidence Matrices.
5. Primitive network-Impedance form and admittance form,
6. Formation of Y Bus by Singular Transformation.
7. Y bus by Inspection Method.
8. Illustrative examples.

1.1 Introduction

- In Power System Analysis–2, we deal with the mathematical modeling and analysis of large electrical power networks
- We represent a complex power system as a network of buses and branches

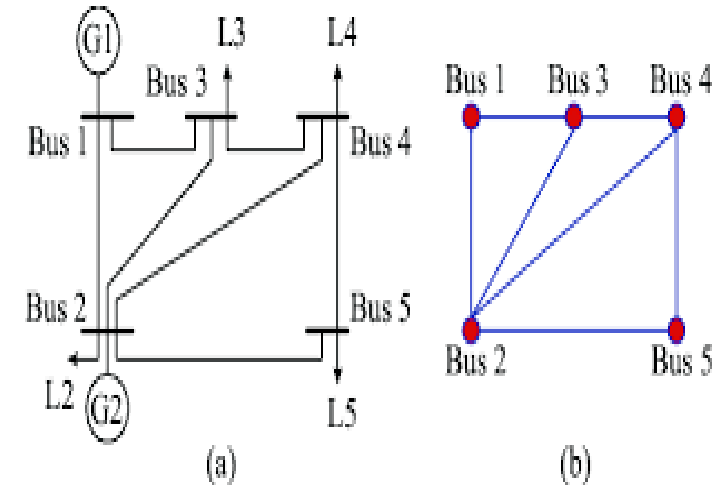
Analogy: Google Maps shows a city as a system of roads and junctions.



Power Network

- Practically: Every generating station, transformer substation, and load center in the power grid acts as a **bus (node)**
- Transmission lines or cables - **branches (links)**
- To analyze such systems, we convert physical components like transformers, lines, and loads into mathematical models and use matrix methods like:

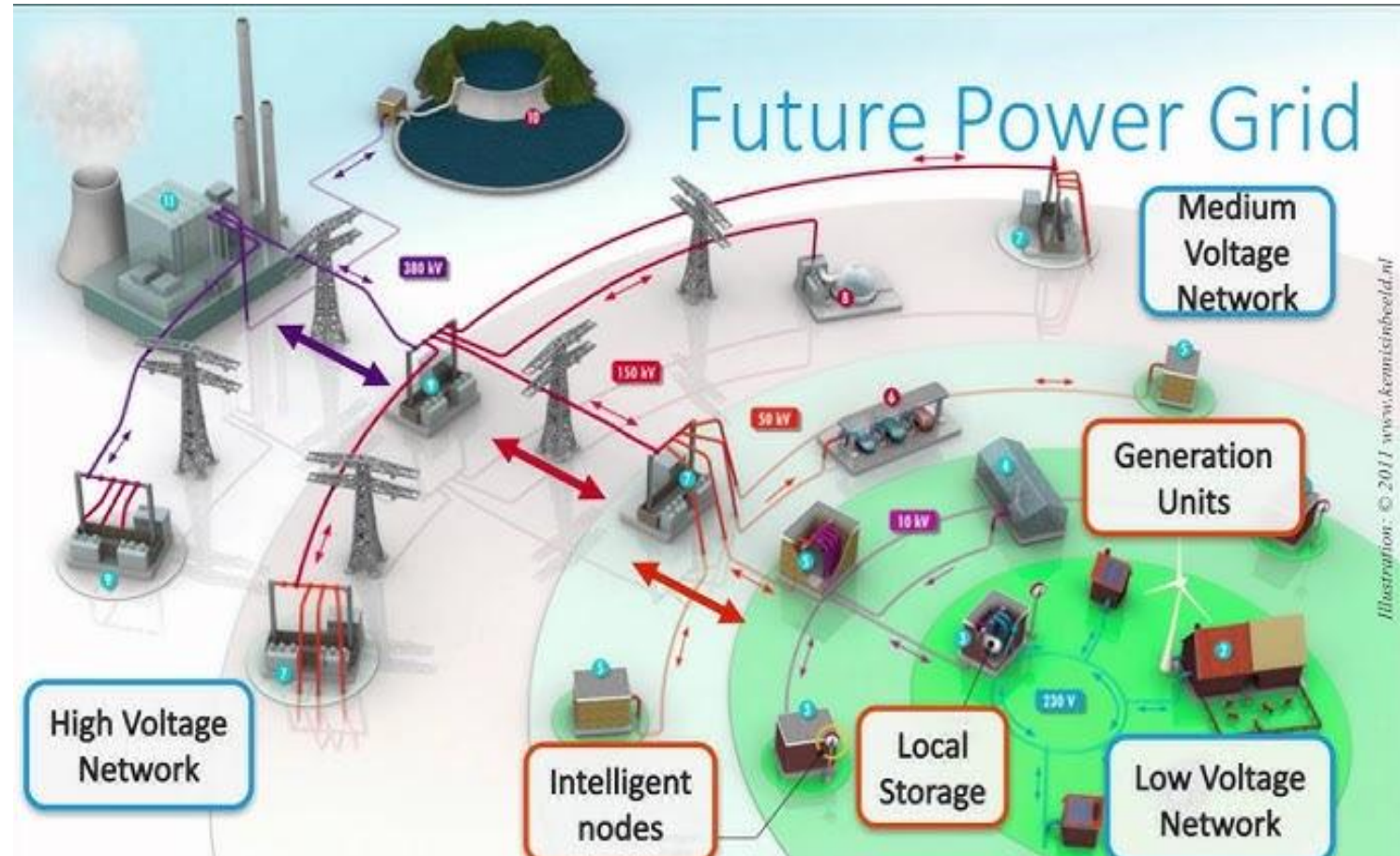
- ✓ **Bus Incidence Matrix (A)** – to show how elements connect to buses
- ✓ **Bus Admittance Matrix (Y-bus)** – to understand how currents and voltages flow in the network



Why Power System Analysis is important?

This is essential in real life for:

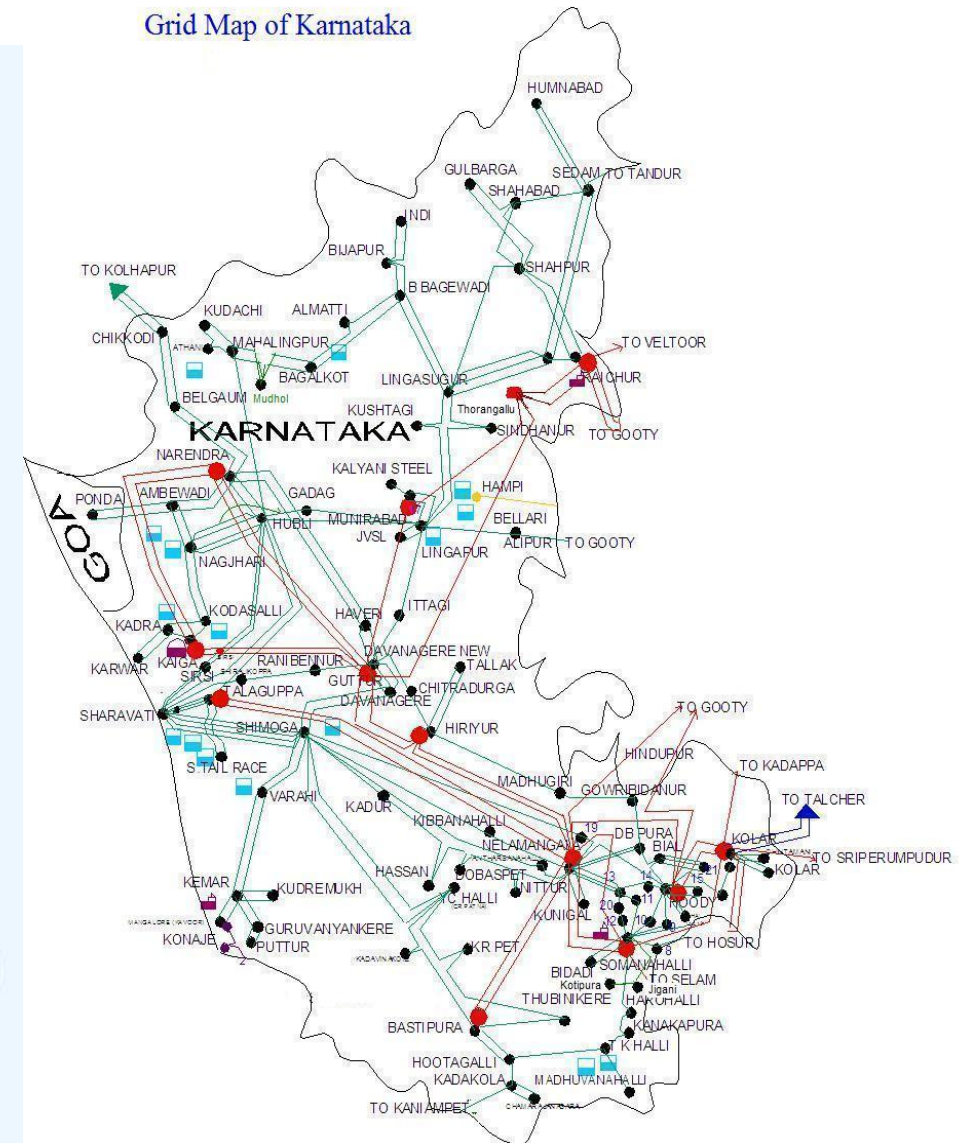
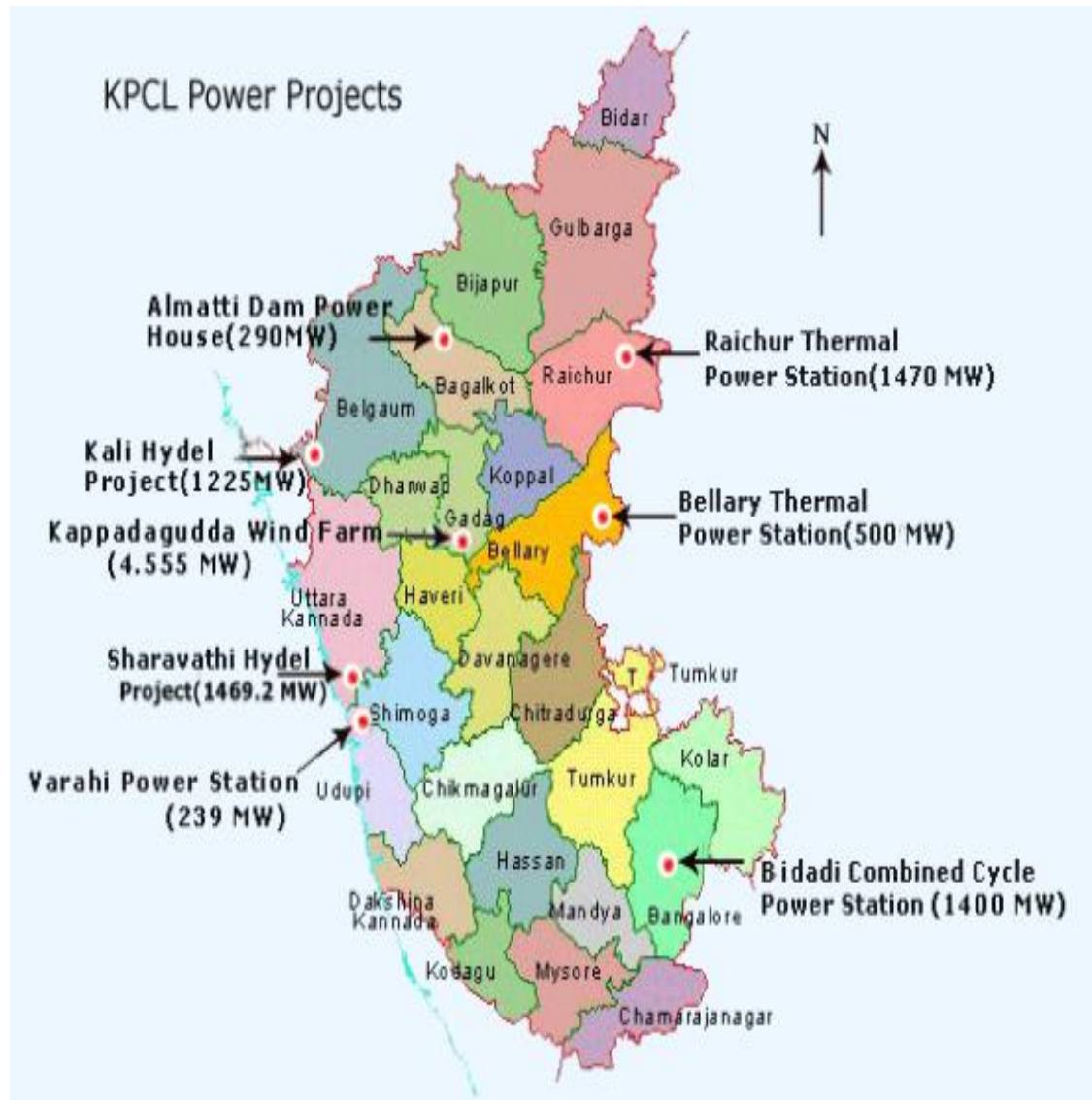
- Power flow studies (what happens when load increases)
- Fault analysis (what happens during short circuits)
- Stability checks (can the grid survive a disturbance?)



- Engineers in control centers monitor & control the power flow using these models.
- During real-time operation, software like SCADA or PSSE (Power System Simulator for Engineers)-uses Y-bus and power flow equations.
- Helps predict overloading and plan upgrades to prevent blackouts.



- Imagine Karnataka's grid is receiving power from multiple sources (Sharavathi hydro, Raichur thermal, solar parks).
- Each one is a bus in your system.
- If one line fails, engineers use the Y-bus model to recalculate how power flows through alternate routes — instantly!



- Graph:** Geometrical interconnection of elements of a network

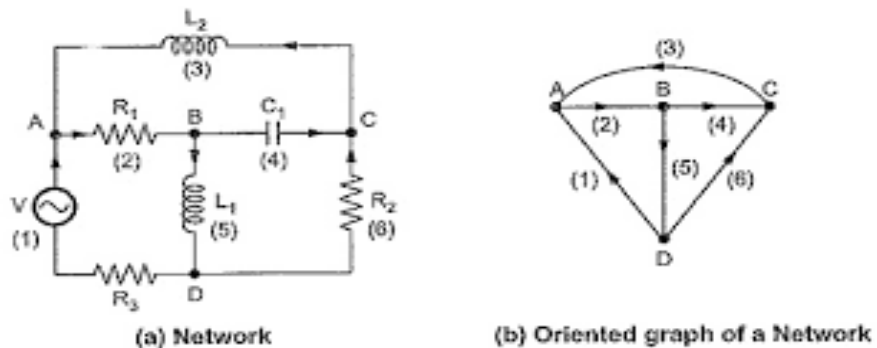
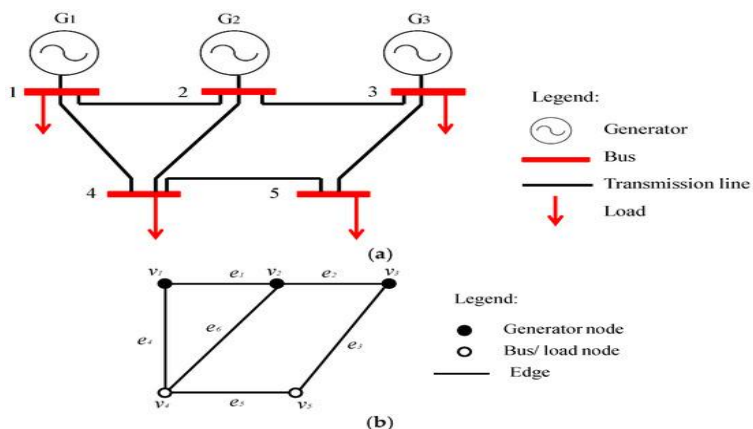
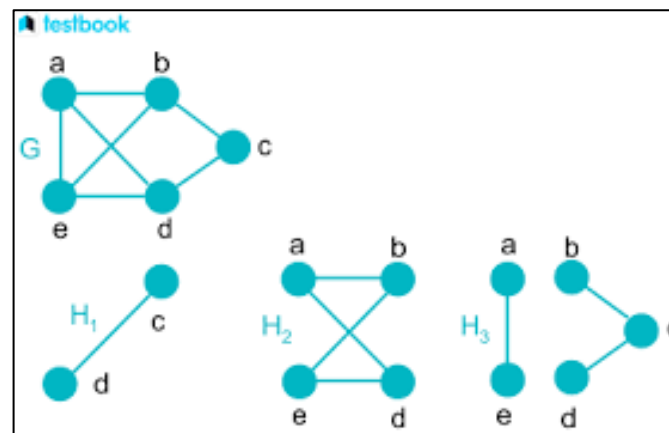


Fig. 5.2 Network and oriented graph



Subgraph: Smaller part of larger subgraph

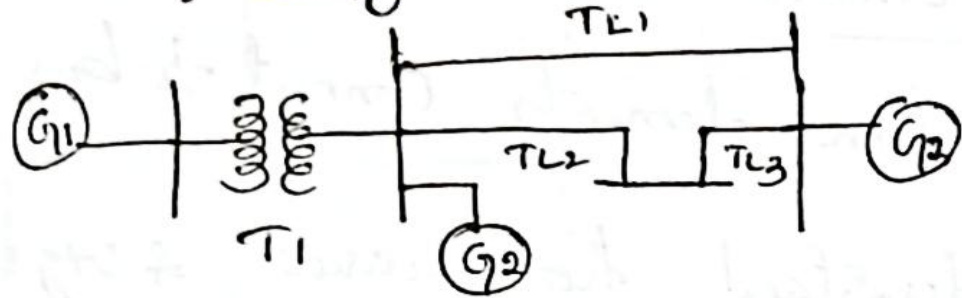


Path: It is a sequence of lines and nodes that connect two buses without repeating any node.

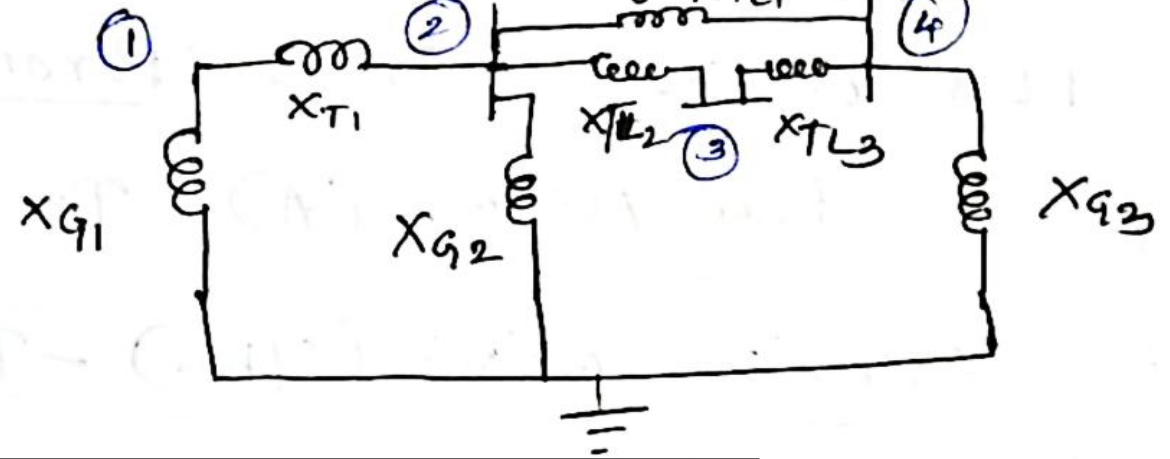
Oriented Graph: If each element of connected graph is assigned with a direction, then it is oriented graph.

1.3 Consider the following: 1.

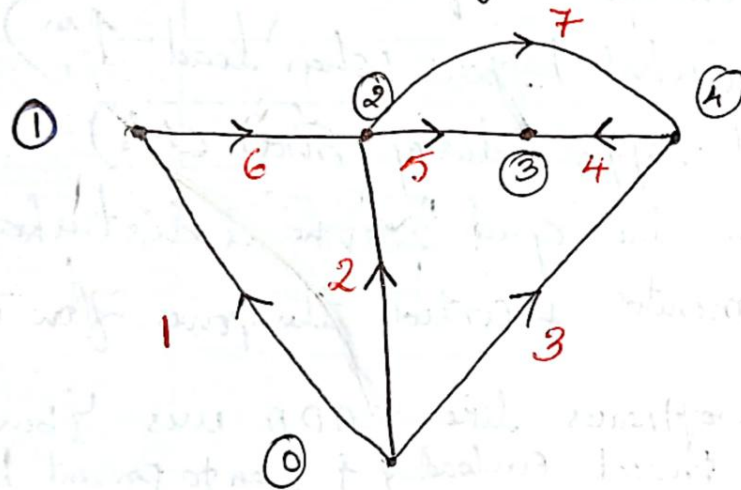
(1) Single line diagram



(2) Reactance diagram of fig (1)



(3) Oriented graph.



Let,

n - no. of nodes = 5

e - no. of lines or elements = 7

b - no. of branches

l - no. of links

(2)

1.4 Tree & Co-tree of the Oriented graph:

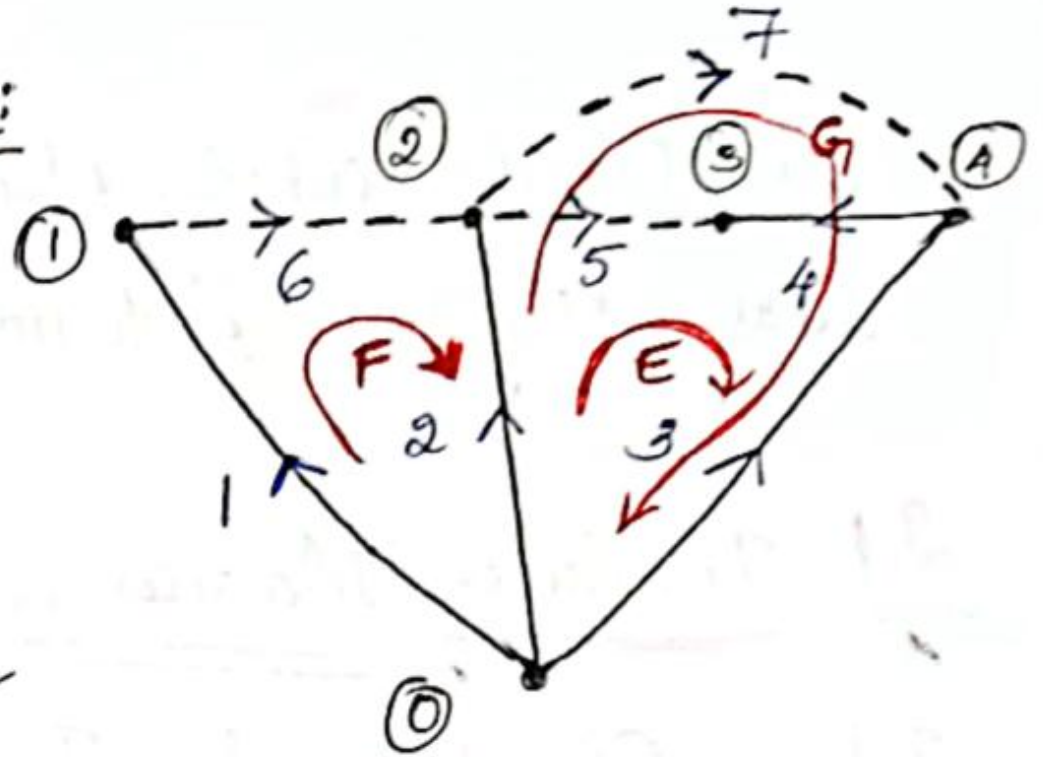
* Tree: A connected subgraph containing all nodes of a graph but no closed path.

* Branch: The elements of a tree are called branches & form a subset of the elements of the connected graph.

We have $b = n - 1$; where n - no. of nodes = 5

$$\underline{\underline{b = 5 - 1 = 4}}$$

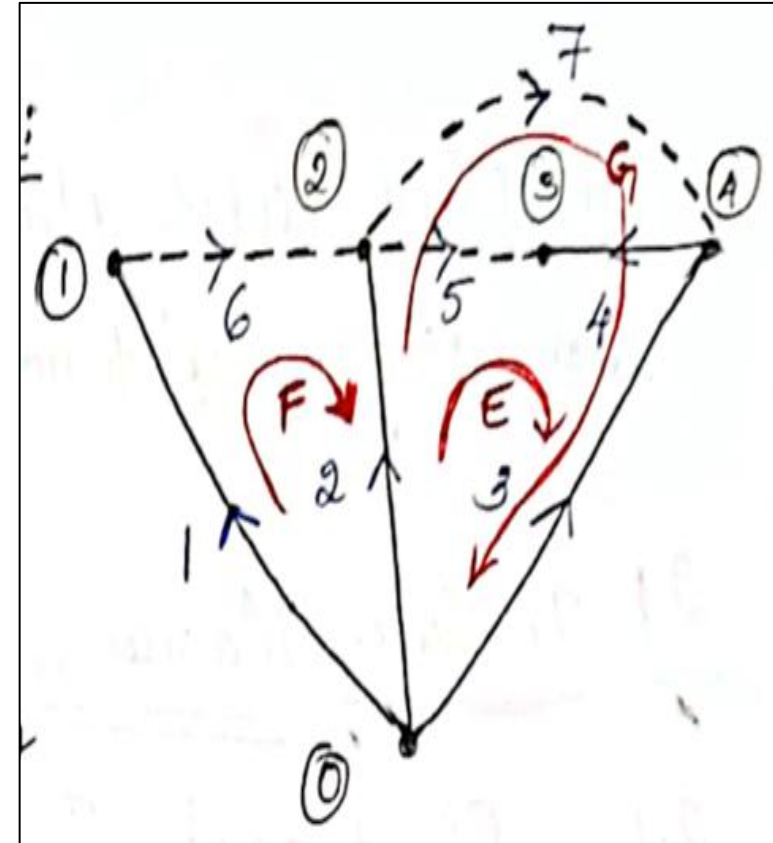
$\Rightarrow$ Branches are 1, 2, 3, 4 (solid lines)



* Links: Those elements of the connected graph that are not included in the tree are called links & forms a subgraph

we have $l = e - b$ or $l = e - (n - 1)$ or $l = e - n + 1$

where e - no. of elements.
 $\Rightarrow l = 7 - 4 = 3$
 Links are 5, 6, 7 (dotted lines)



* Co-tree: It is complement of tree. Links & co tree are same.

* Loop: It is a closed path in a graph. It starts & ends at the same node. If a link is added to the tree, the resulting graph contains one closed path.

* Basic loop: The loops which contain only one link are independent & are called basic loops.

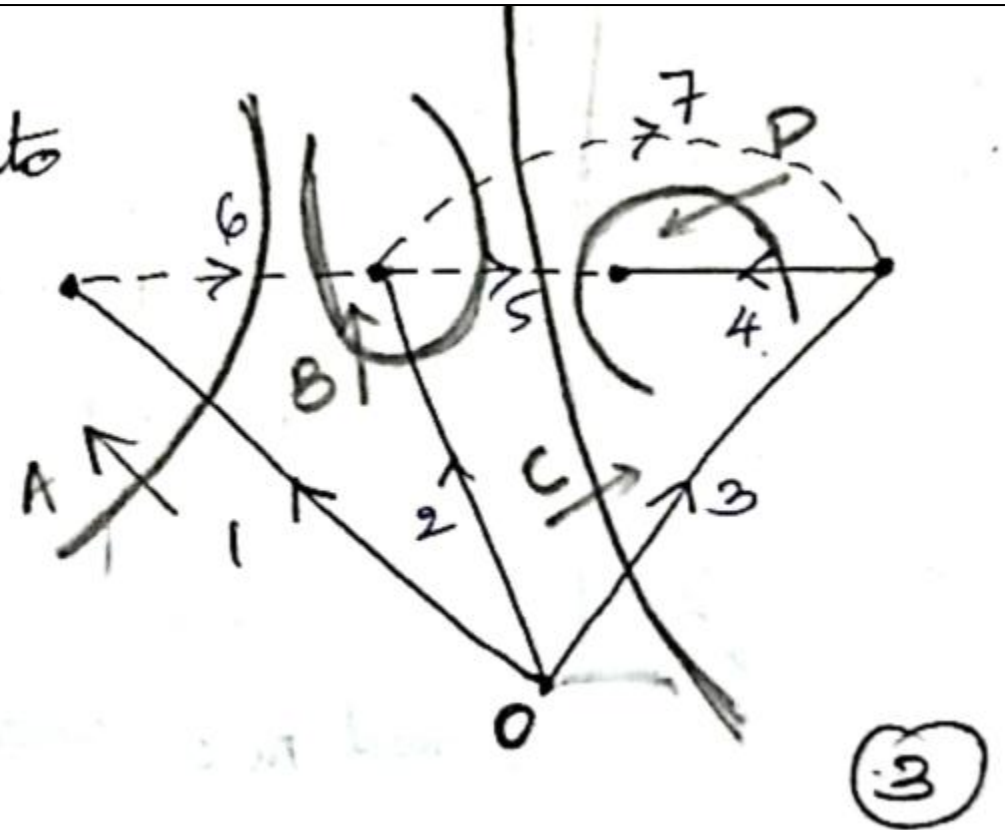
* No. of loops = No. of links

* Orientation of a basic loop is chosen - same as that of link

* Cut-Set: It is a set of elements that, if removed, divides a connected graph into two connected subgraphs.

* Twig: Elements of a tree are called twigs or tree-branches.

Direction of cut-set is same as that of branches



* Basic Cut set: Cutsets which contain only one twig & remaining links. They are equal to no. of twigs.

2. Incidence Matrices:

2.1 Element- node Incidence Matrix or Branch node incidence matrix or bus incidence matrix ' A' '= $(e \times n)$

* The incidence of elements to nodes in a connected graph is shown by element-node incidence matrix

* The elements are obtained as follows.
e - are placed on rows ; n - are placed on columns in matrix.

$a_{ij} = 1$ $\rightarrow$ If i^{th} element is incident to j oriented away from node

$a_{ij} = -1$ $\rightarrow$ If i^{th} element is incident to j oriented towards the node

$a_{ij} = 0$ $\rightarrow$ If i^{th} element is not incident to j^{th} node.

* Dimension of matrix is $\hat{A} = e \times n$

7

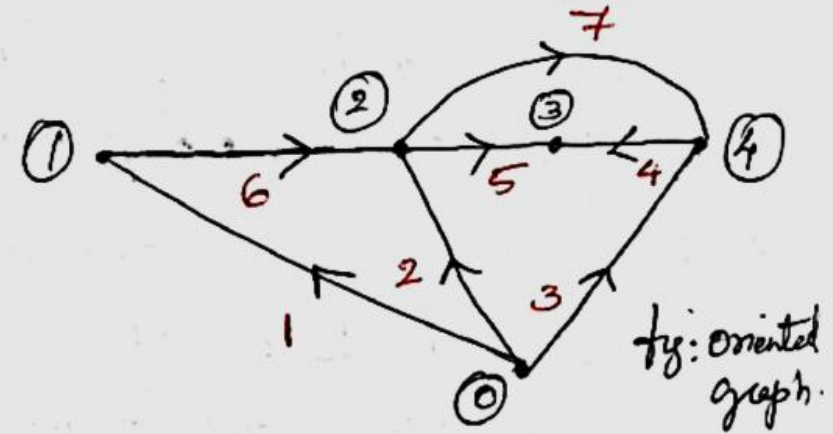
* Dimension of matrix is $\hat{A} = e \times n$

* $\hat{A}$ for oriented graph in 1.3 is

elements ↓

e \ n	Nodes	①	②	③	④
1		1	-1		
2		1		-1	
3		1			-1
4				-1	1
5			1	-1	
6			1	-1	
7			1		-1

Element node incidence matrix - $\hat{A}$



②

	①	②	③	④
1	-1			
2		-1		
3			-1	
4				-1
5		1	-1	
6		1	-1	
7		1		-1

entering leaving

2.1 Bus Incidence Matrix

* Any nodes of connected graph can be chosen as reference node of the Column corresponding to that node is deleted from $\hat{A}$. The matrix so formed is called the Bus-incidence matrix A .

* Dimension of $A = e \times (n-1)$; where $(n-1)$ - rank of matrix.

* From above example consider the node ⑥ as reference node.

We get matrix A given by,

$$A = \begin{matrix} & e \backslash n & \textcircled{1} & \textcircled{2} & \textcircled{3} & \textcircled{4} \\ \begin{matrix} i \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{matrix} & \left[\begin{array}{cccc} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & -1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{array} \right] \end{matrix} = \begin{matrix} e \backslash n \\ \left[\begin{array}{c} \text{Branch} \\ \text{Link} \end{array} \right] \left[\begin{array}{c} A_b \\ A_l \end{array} \right] \end{matrix}$$

* Above matrix is partitioned into 2 submatrices as shown above:

$$A_b = b \times (n-1)$$

$$A_l = l \times (n-1)$$

2.2 Branch- path Incidence matrix ' k ' = ($b \times \text{paths}$)

* The incidence of branches to paths in a tree is shown by the branch-path incidence matrix, where a path is created from bus to reference node.

* Elements of the matrix are:

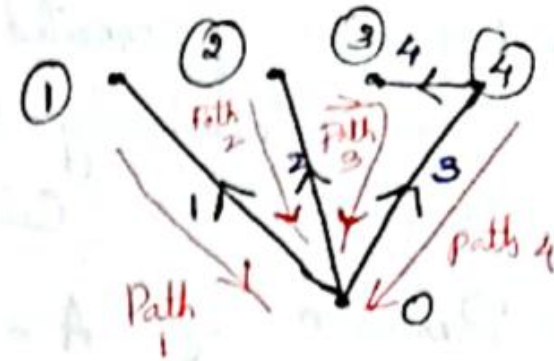
- $K_{ij} = 1$ $\rightarrow$ If the i^{th} branch is in the path from j^{th} bus to reference & is oriented in same direction.
- $K_{ij} = -1$ $\rightarrow$ If the i^{th} branch is in the path from j^{th} bus to reference but oriented in opposite direction.
- $K_{ij} = 0$ $\rightarrow$ If the i^{th} branch is not in the path from j^{th} bus to reference.

* $n = 5$, $b = n - 1 = 5 - 1 = 4$. Where b - branch.

* $k = b \times \text{path}$.

$K =$

b \ path	①	②	③	④
1	-1	0	0	0
2	0	-1	0	0
3	0	0	-1	-1
4	0	0	-1	0



* Branch-path incidence matrix & submatrix A_b relates the branches to paths & branches to buses resp.

$$\therefore A_b \cdot K^T = U$$

$$\therefore K^T = [A_b]^{-1}$$

2.3 Basic Cut-set Incidence matrix '**B**'=(**e**×**b**) cntd.

* The incidence of elements to basic cut sets of a connected graph is shown by the basic cut-set incidence matrix '**B**'.

* The elements of the matrix are:

$b_{ij} = 1$ → If i^{th} element is incident to & oriented in the same direction as the j^{th} basic cut-set

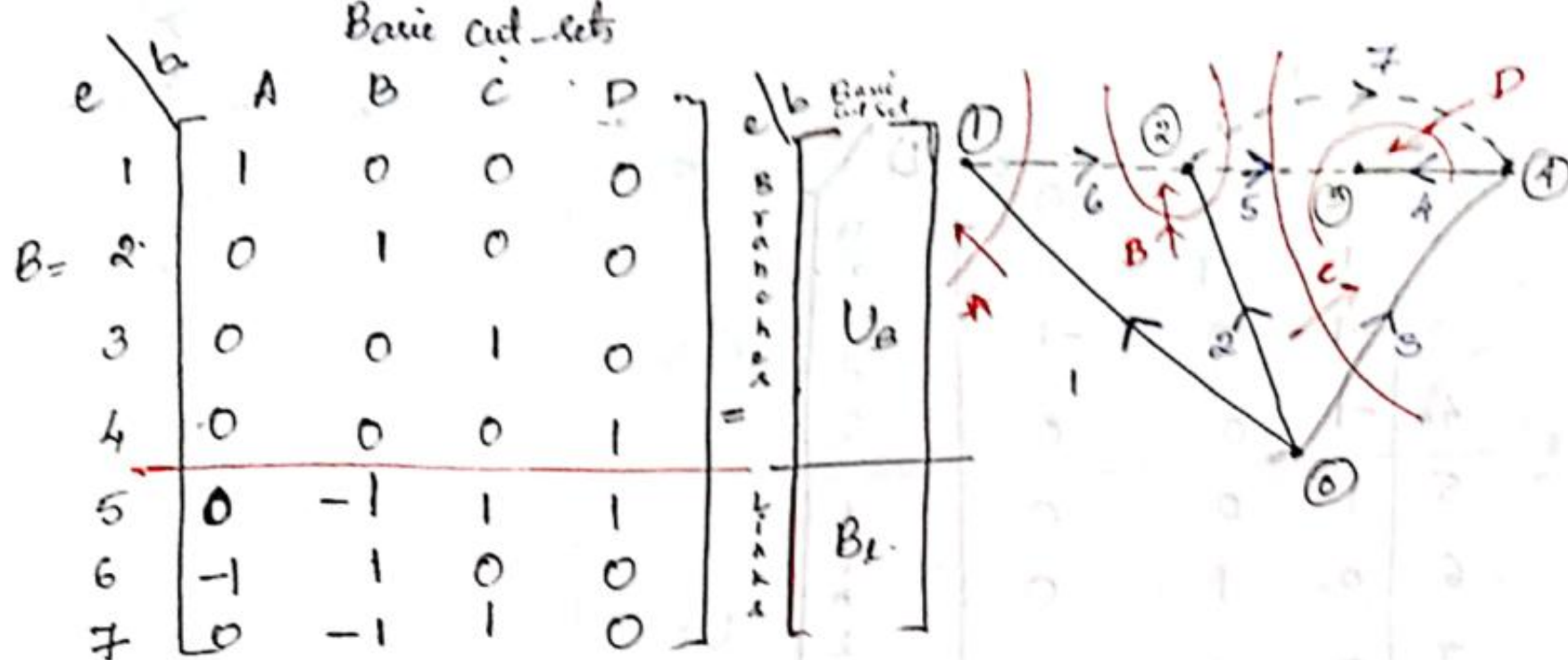
$b_{ij} = -1$ → If i^{th} element is incident to & oriented in opposite direction as the j^{th} basic cut set

$b_{ij} = 0$ → If i^{th} element is not incident to j^{th} basic cut set

* Basic cut-set incidence matrix dimension is $B = e \times b$.

* Consider the cut-set graph shown previously.

2.3 Basic Cut-set Incidence matrix ' $B=(exbc)'$ ' cntd.



Here $U_B \rightarrow$ Identity Matrix $\rightarrow$ Corresponds to branches & cut-sets.

$B_L \rightarrow$ Can be obtained from Bus incidence Matrix as follows.

$$B_L \cdot A_b = A_L$$

$$B_L = A_L \cdot A_b^{-1}$$

$$\text{WKT.} \cdot A_b^{-1} = K^T$$

$$\therefore \boxed{B_L = A_L \cdot K^T} \rightarrow B_L \text{ intn of Bus incidence matrix.}$$

- * The incidence of elements to basic loops of connected graph is shown by basic loop incidence matrix C
- * The elements of the matrix are:
 - $C_{ij} = 1 \rightarrow$ If the i^{th} element is incident to & oriented in the same direction as j^{th} basic loop.
 - $C_{ij} = -1 \rightarrow$ If the i^{th} element is incident to & oriented in opposite direction as j^{th} basic loop.
 - $C_{ij} = 0 \rightarrow$ If the i^{th} element is not incident to j^{th} basic loop.
- * The basic loop incidence matrix dimension is $C = e \times l$.

